Program Information								
	[Lesson Title] ***Salaries, Interest, and Inflation***				**TEACHER NAME**		**PROGRAM NAME**	
	[Unit Title] ***Algebra and Patterns***				**NRS EFL(s)** **3 – 6**		**TIME FRAME** **120 minutes**	

Instruction								
	ABE/ASE Standards – Mathematics							
	Numbers (N)		**Algebra (A)**		**Geometry (G)**		**Data (D)**	
	Numbers and Operation	**N.3.26**	Operations and Algebraic Thinking		Geometric Shapes and Figures		Measurement and Data	
	The Number System	**N.4.6**	Expressions and Equations	**A.3.9** **A.3.15** **A.4.7** **A.4.8** **A.5.2**	Congruence		Statistics and Probability	**D.4.1**
	Ratios and Proportional Relationships		Functions	**A.4.15** **A.6.6** **A.6.9**	Similarity, Right Triangles. And Trigonometry		**Benchmarks identified in *RED* are priority be benchmarks. To view a complete list of priority benchmarks and related Ohio ABLE lesson plans, please see the Curriculum Alignments located on the Teacher Resource Center (TRC).**	
	Number and Quantity	**N.5.1**			Geometric Measurement and Dimensions			
					Modeling with Geometry			

Mathematical Practices (MP)				
✓	Make sense of problems and persevere in solving them. (MP.1)	✓	Use appropriate tools strategically. (MP.5)	
✓	Reason abstractly and quantitatively. (MP.2)	✓	Attend to precision. (MP.6)	
✓	Construct viable arguments and critique the reasoning of others. (MP.3)	☐	Look for and make use of structure. (MP.7)	
☐	Model with mathematics. (MP.4)	☐	Look for and express regularity in repeated reasoning. (MP.8)	

LEARNER OUTCOME(S)

- Students will choose between the mean, median, and mode in financial situations, and then will justify their decision.
- Students will compare options based on the exponential growth of money.

ASSESSMENT TOOLS/METHODS

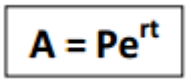
- Steps 9 and 10 will serve as evidence of student mastery.
  - During Step 10, the teacher should actively listen to partner discussions for signs of understanding or of misconceptions. If students are working alone, the teacher should have students speak out loud as they solve the problem.
  - During Step 11, allow students the opportunity to modify their solutions based on what they learn from watching others present their solutions.
- Exit Slip: 1. Using interlocking cubes, find the mean of the following set of numbers: {3, 2, 3, 6, 7, 3, 4} 2. If you invest $500 in a 5-year Certificate of Deposit with a base rate of 5%, how much will you receive at the end of the five years?

LEARNER PRIOR KNOWLEDGE

- Students should be able to perform accurate calculations for exponential equations using order of operations.
- Students should be able to plot a coordinate pair on the X-Y plane, and then interpolate between points.

	INSTRUCTIONAL ACTIVITIES	RESOURCES
	1. Review. a. Write the following numbers on the board: 1, 4, 4, 3, 3, 7, 5, 4, 5. Ask students to write down the mean, median, mode, range, five-number summary, and interquartile range (IQR) of this set. If there are any terms that the students are not familiar with, give them the definition from the *Vocabulary Sheet*. 2. Pass out 50 interlocking cubes to each student. To get them familiar with using the cubes, ask them to model the measures of center from Step 1. In other words, how could they use the cubes to figure out the mean, median, and mode of the set? There are two main types of methods the students might use (see *Teacher Answer Sheet*, Method 1 and Method 2); make sure they see them both (if they only come up with one, show them the other one also). a. Method 1: Each cube represents 1 unit. Using this method, the students will actually need 36 cubes to model the problem. To find the median and mode, they must first arrange their stacks in order of height. To find them mean, take cubes from the tallest stacks and place them onto the smaller stacks until all the stacks are as equal as possible (for this problem, they will all equal 4). b. Method 2: Each cube represents 1 number. In this case, they will only need 9 cubes, but they will need some way to hold a place for each value, even the missing values of 2 and 6. Using this method, the mode is the highest stack, the median is the 5th cube from the left, and they can find the mean by taking a cube from any two stacks and placing both cubes in the middle. For example, they could take one cube from the 7 stack, and one cube from the 3 stack, and place them both on 5. Continuing in this manner, all cubes will eventually end up on 4, which is the mean.	Interlocking Cubes for student use Calculators for student use (if none are available, you can leave exponential equations in unreduced form) Student copies of *Task 1: Expected Salaries* handout (attached) Student copies of *Task 2: Paying Bills and Inflation* handout (attached) Student copies of *Task 3: Savings Accounts* handout (attached) *Teacher Answer Sheet* (attached) *Vocabulary Sheet* (attached)

	3. (I do) Teacher models the solution process. Pose this problem to students, “Change the original set of numbers, so that (a) the median stays the same, (b) the mode stays the same, and (c) the mean decreases.” Use the talk aloud technique as you decide which method you want to use (either one is fine), and work your way through the problem. You may want to start with the mode and ask yourself out loud what it means to “keep the mode the same” (in other words, you can’t change any of the 4’s). When you feel you have an answer, make sure you go back through and check whether the median, mode, and mean of your modified set meet the requirements of the problem. Be sure to comment that this is not the only correct solution. 4. (We do) Teacher and students collaboratively work through the problem. Pose this problem to students, “Change the original set of numbers, so that (a) the median decreases, (b) the mode stays the same, and (c) the mean increases.” Allow students to choose a starting point for the problem, but if they struggle to decide, suggest that they begin with the mode again. The key learning point in Step 4 is that the mean is affected by how far each block is from the center, whereas the median is only impacted by whether a block is to the right or the left of the center. 5. (You do) Students independently work through the problem. Pose this problem to students, “Find a set of five numbers for which (a) the median is 3, (b) the mode is 3, and (c) the mean is 5.” Make sure that students check their answer against all three criteria when they finish. After everyone is done, have students share with the group how they approached the problem. 6. (You do) Students independently work through the problem. Pose this activity to students, “Start with the set { 1, 1, 2, 3, 8 }. Find the mean, median, and mode.” (pause, and allow students to work this	

	out; compare answers) “Now, double every number in the set and find the new mean, median, and mode.” (pause, work, compare) “Then, double every number in the new set and find the new mean, median, and mode.” (pause, work, compare) “Based on these findings, make a conjecture about how doubling any set affects the mean, median, and mode of the set.” Make sure students come up with the rule that doubling each number of a set also doubles the mean, the median, and the mode. Then tell students that this is true for any exponential increase or decrease (e.g., multiplying each number by 1.4 causes the mean, median, and mode to increase by 1.4 and multiplying each number by .5 causes the mean, median, and mode to be split in half). 7. Introduce the context. Tell students that the three tasks will deal with common financial scenarios where measures of center are useful. However, as the tasks will show, it is important to understand the differences between the measures of center in order to interpret situations accurately. Another important concept in financial matters is exponential growth; two major examples are inflation and interest rates. Since growth is often constant, we will use the formula for continuous compounding in both of these types of situations.  where t = the amount of time, r = the base rate of increase per 1 unit of time, P = principal (the initial amount of money), A = Amount (the final amount of money after t units of time), and e = Euler’s number (2.7182…). Another related formula is the one for calculating annual percentage yield (APY).	

	$APY = e^r - 1$ Many banks and loan organizations publish both the base interest rate and the APY. The APY is the actual interest rate when the base rate is compounded throughout the period of time. 8. (I do) Teacher models the solution process through the *Task 1: Expected Salaries* handout. Except for Question #6, these are basic calculation questions and conceptual questions. Although you can discuss the conceptual ones with students to some extent, remember that you are the leader for this step, and so you should be the one doing most of the talking as work through these problems. Before you do the calculations for Question #6, make a prediction out loud, based on the relationship you uncovered in Step 6 and the value for the range that you found in Question #1. A = Pert APY = er – 1 9. (We do) Teacher and students collaboratively work through the *Task 2: Paying Bills and Inflation*. Allow students to take the initiative on the first question, but correct them immediately if you see them making a mistake. On Question #2 and #3, ask if anyone has any ideas before you give an explanation; make sure they justify their suggestions instead of just guessing wildly. If students seem completely puzzled, have them get out the interlocking cubes and work through a much simpler version of the same problem (for example, model a set of {2, 6, 7}, and then ask if it would be better to pay $2, $6, and $7 or the mean of $5 three times? Make sure that you choose a set where the mean and median are not equal, or else the students may get a false idea for Question #3). When you plot the graph in #4, make sure that the exponential curve pattern is clear (i.e., the relationship is not a straight line).	

	10. (You do) Students independently work through the *Task 3: Savings Accounts*. Depending on your class dynamics, either partner students together or have them work individually. Before you pass out the task, explain that you want the students to tackle this problem as independently as possible. After passing out the handouts, walk around the room silently monitoring the students' progress. When you see them run into difficulties, try not to answer their questions directly; instead, remind them of similar situations from the first two tasks. On #4, make sure they don't try to solve for an exact answer, as this is a very complicated math problem. Instead, they can just look at where the lines on their graph intersect. 11. Have each student (or pair) share both the process they used and their final comparisons. When students disagree, do not immediately provide the correct answer; allow each student or pair to try to convince the other first.	
	DIFFERENTIATION	

Reflection	**TEACHER REFLECTION/LESSON EVALUATION**
	ADDITIONAL INFORMATION **NEXT STEPS** Have students take a look at either loans they are paying on, or savings they are accruing. They should find out the base interest rate (or APY) and calculate how much the interest will add up to over the span of 10 or 20 years. **TECHNOLOGY INTEGRATION** The site below offers explanations and actual figures for inflation rates over time. It also builds on inflation data with other financial concepts and data for students who are interested in exploring the topic further. http://inflationdata.com/ The National Library of Virtual Manipulatives provides many exploratory tools. On this page, students can explore the "Loan Calculator" and the "Savings Calculator," for a more in-depth look at this lesson's topics. http://nlvm.usu.edu/en/nav/category_g_4_t_5.html **PURPOSEFUL/TRANSPARENT** The fact that the value of money can change over time is both intriguing and confusing to many students. This lesson starts with simple visualizations (interlocking cubes), and progresses on to critical everyday situations such as salaries, bills, interest, and inflation. **CONTEXTUAL** This lesson centers on the concept that the value of money is constantly changing – both the money we have and the money we owe. For any member of society (presumably all the ABLE learners), this is an important topic for gaining financial understanding and, ultimately, control. **BUILDING EXPERTISE** Students will build on their simple understanding of calculating the mean, median, and mode to understanding how each of them are affected by individual data points. Moreover, they will learn to make decisions based on this understanding of central tendency.

	Using interlocking cubes and their understanding of central tendency, they move on to realize the power of exponential functions over time.

NOTE: The content in the Additional Information box exceeds what is required for the OBR Approved Lesson Plan Template. This information was provided during the initial development of the lesson, prior to the creation of the OBR Approved Lesson Plan Template. Feel free to remove from or add to the Additional Information box to suit your lesson planning needs.

Task 1: Expected Salaries

Joe has received an associate's degree and is now looking for a workplace. He is considering three companies--Company A, Company B, and Company C. Each of the companies has published some salary information about their staff (See Table 1). Use this table to answer the following questions.

Table 1
Staff Salaries at Three Companies in 2010 (in thousands of dollars per year)

	Company A	Company B	Company C
Mean	36	38	35
Q0	24	21	25
Q1	26	25	31
Q2	30	27	33
Q3	37	30	37
Q4	45	60	50

1. What is the median salary, range, and interquartile range (IQR) for each company?

2. In each of the companies, the median salary is lower than the mean salary. What does this indicate?

3. Create a graph that effectively displays and compares salaries from the three companies.

4. In which company does the management probably get paid the best? Justify your answer.

5. Why could it be misleading for the companies to only publish the mean salary?

6. If raises in Company A are calculated at a constant 3% rate, what would the *range* of salaries be for Company A in 2020? (Note: You do not have to calculate every quartile.) What would the mean salary be?

Vocabulary Sheet

Annual Percentage Yield – the effective interest rate that accounts for compounding over the course of a year.

Conjecture – an educated guess based on incomplete evidence.

Exponential – numbers or quantities that are raised to an exponent.

Five Number-Summary – Q0, Q1, Q2, Q3, and Q4

Interpolate – estimate the y-values for all x's in between the minimum and maximum x-values.

Interquartile Range (IQR) – Q3 minus Q1

Mean – the sum of a set of values, divided by the number of elements in the set.

Median – the middle number in an ordered set (if the number of elements is odd) or the mean of the two middle numbers of an ordered set (if the number of elements is even).

Mode – the element(s) in a set which occur with the highest frequency.

Quartiles – Q0 is the smallest number in a set, Q2 is the median of the set, and Q4 is the largest number in the set. Q1 is the median of all the numbers before Q2 (and not including Q2) in the ordered set. Q3 is the median of all the numbers after Q2 (and not including Q2) in the ordered set.

Range – Q4 minus Q0

Task 2: Paying Bills and Inflation

Roberta has been receiving offers in the mail to enroll in a fixed-rate billing program. She has collected her monthly bill totals in a spreadsheet (See Table 2) and is now reviewing the offers. Use this table to answer the following questions. Assume that she will continue to use roughly the same amount of services in the following year as she did in the past year.

Table 2

Monthly Bills for Various Services in 2011

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Electric	40	43	45	48	55	63	75	78	70	46	44	43
Gas	65	60	66	48	20	24	23	18	22	24	38	55
Water/Sewer	15	30	15	15	15	30	15	15	30	15	15	15
Cable/Internet	55	55	55	55	55	55	55	55	55	55	55	55

1. What is the five-point summary, range, and interquartile range (IQR) for each of the four types of services?

2. Offer 1 allows Roberta to pay the mean price for each service each month. Without calculating, decide whether she would save money with Offer 2 (in comparison with her current payment plan). Justify your answer.

3. Offer 2 allows Roberta to pay the median price for each service each month. Without calculating exact totals, estimate whether she would save money with Offer 1 (in comparison with her current payment plan). Justify your answer.

4. Roberta would like to estimate her expected bills for the next 10 years. Assume that there is a constant 15% inflation rate and that her expenses for these services in 2011 totaled about $2500. Estimate her bills for every two years (2013, 2015, 2017, 2019, and 2021) and then plot the totals for all six years (including 2011) on a graph.

Task 3: Savings Accounts

Jackson currently has a savings account at Seventh National Bank, but has been checking out other options. Use the information provided in Table 3 to answer the below questions. All three banks shown in Table 3 use continuous compounding (in other words, the rates shown are not the APY).

Table 3

Interest Rates and Signup Bonuses at Three Banks

	Bank A	Bank B	Bank C
Savings Account Interest Rate	2.00%	3.00%	0.50%
Signup Bonus	$20.00	$0.00	$40.00

1. If Jackson opens up an account with $1000 and does not deposit or withdraw for one year, what is the mean amount of money he would earn? (the mean of the interest plus signup bonus for the three banks)

2. What is the APY for each of the three banks? Remember that APY = $e^r - 1$.

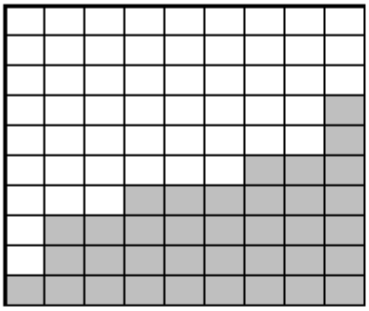
3. Make a single graph showing how much money Jackson would earn on his initial deposit of $1,000 over the course of three years with each of the three banks. (Plot all three lines on a single graph; assume that he does not withdraw or invest any more money.) Why do the relationships look like a straight line instead of following an exponential curve?

4. Estimate how long Jackson would have to invest his money before Bank B would provide the greatest overall savings. (Hint: Don't actually *calculate* this; use the information from #3 to estimate!!) Explain why, for any times greater than this (assuming all rates stay the same), Bank A and Bank C would never again "catch up" to Bank B.

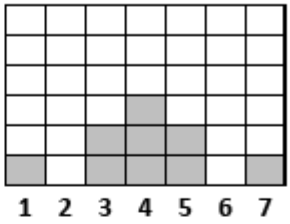
Salaries, Interest, and Inflation: Teacher Answer Sheet

From the Lesson Plan:

Step 2



Method 1



Method 2

Expected Salaries Task Answers

1. Company A: median = $30,000; range = $21,000; IQR = $11,000

Company B: median = $27,000; range = $39,000; IQR = $5,000

Company C: median = $33,000; range = $25,000; IQR = $6,000

2. This suggests that the salaries are not equally distributed; in other words, more than half of the employees earn less than the mean salary. (Simply put: The high management salaries are distorting the mean.)

3.

4. Company B. Management would likely be the highest few salaries in the company and Company B has the highest maximum salary (plus, because Company B has the highest mean and yet is less than the other two companies on Q0, Q1, Q2, and Q3, we know that the top 25% of salaries must be very high).

5. In an asymmetric distribution (like with salaries), the mean is not representative of the typical salary.

6. Range = $21e^{.03*10} = 28.3$. In other words, the range will be about $28,300 in 2020.

Paying Bills and Inflation Task Answers

1. Electric: Five-point summary = {40, 43.5, 47, 66.5, 78}, range = 38, IQR = 23

Gas: Five-point summary = {18, 22.5, 31, 57.5, 66}, range = 48, IQR = 35

Water/Sewer: Five-point summary = {15, 15, 15, 22.5, 30}, range = 15, IQR = 7.5

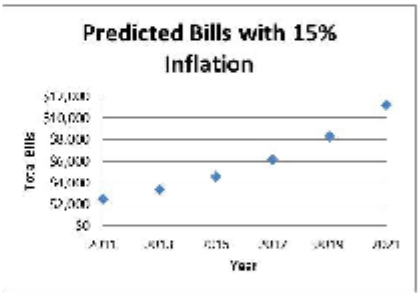
Cable/Internet: Five-point summary = {55, 55, 55, 55, 55}, range = 0, IQR = 0

2. Paying the mean price each month would be the same as paying the actual price each month (as long as you ignore inflation).

3. Clearly, Cable/Internet would be the same for either mean or median. For Water/Sewer, the median of 15 is certainly less than the mean. For Gas and Electric, we can estimate that the median would be less than the mean because Q2 - Q0 is less than Q4 - Q2 in both cases. In other words, because the median is closer to the minimum than the maximum, we can guess that the mean will be higher than the median. In conclusion, Offer 2 seems to be a good deal for Roberta.

4.

Year	Bills
2011	$2,500
2013	$3,375
2015	$4,555
2017	$6,149
2019	$8,300
2021	$11,204



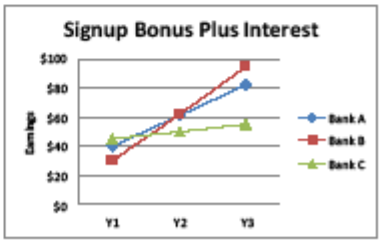
Savings Account Task Answers

1. Startup + 1 yr interest = $40.20 (Bank A), $30.45 (Bank B), $45.01 (Bank C) leads to an average of $38.56.

2. 2.020% (Bank A), 3.045% (Bank B), 0.501% (Bank C)

3. Actually, they are exponential, but because the interest rate is so small, the first few years will appear to be in a straight line.

	Y1	Y2	Y3
Bank A	$40	$61	$82
Bank B	$30	$62	$94
Bank C	$45	$50	$55



4. Even by the end of the second year, Jackson will have earned the most at Bank B. After this, Bank B will always provide the most total earnings because it has the highest interest rate (and the interest rate is the only thing causing the earnings to increase after the first year).